

LESSONS FROM SCHWINGER EFFECTIVE ACTION FOR BLACK HOLES

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Abstract

We revisit the Hawking radiation by comparing the effective actions in the in-out formalism, and advance an interpretation of the vacuum polarization and the Hawking radiation. The equivalence exists between the spinor QED effective action in a constant electric field and the non-perturbative effective action of a massless boson on the horizon of a Schwarzschild black hole.

1 Introduction

Pair production in strong background fields has been one of the most important issues in theoretical physics since the computation of the one-loop effective action in a constant electromagnetic field by Heisenberg and Euler [1] and Schwinger [2] and the discovery of the black hole radiation by Hawking [3]. The virtual pairs from vacuum fluctuations are separated into real pairs by the strong electric field in the Schwinger mechanism and by the causal horizon of the black hole in the Hawking radiation, as summarized in Table 1.

The pair production is accompanied by the vacuum polarization, that is, the real part of the nonperturbative effective action. In quantum electrodynamics (QED), for instance, the mean number of pairs or the vacuum persistence (twice the imaginary part of the effective action) is closely related to the pole structure of the vacuum polarization. In the in-out formalism based on the Schwinger variational principle, the effective action is the scattering matrix amplitude between the in- and the out-vacua, which can be manifestly realized by the Bogoliubov transformation method [4].

In this talk, we revisit the new approach to the vacuum polarization and the Hawking radiation of a Schwarzschild black hole in analogy with the Heisenberg-Euler and Schwinger effective action in QED [5]. Though it results from quantum field theory at one-loop, not from quantum gravity, the nonperturbative effective action, however, may still shed light on quantum aspects of black holes.

Table 1: Strong Field Physics: Analogy between QED and Black Hole

	Strong QED	Black Hole
External agent	Electric field	Event horizon
Pair production	Schwinger mechanism	Hawking radiation
Nonperturbative action	Vacuum polarization	Stress tensor

2 Schwinger Mechanism and Effective Action

The vacuum polarization and the pair production have been systematically studied in spinor QED by Heisenberg and Euler and in scalar as well as spinor QED by Schwinger. The vacuum polarization may be written as [2]

$$\mathcal{L}_{\text{eff}} = (-1)^{2\sigma} \frac{(1+2\sigma)}{2} \frac{qE}{2\pi} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \mathcal{P} \int_0^\infty \frac{ds}{s} \exp\left(-\frac{m^2 + \mathbf{k}_\perp^2}{2qE} s\right) \times \left[\frac{\cos^{2\sigma}(s/2)}{\sin(s/2)} - \frac{2}{s} + (-1)^{2\sigma} \frac{1-\sigma}{6} s \right], \quad (1)$$

where $\sigma = 0$ for scalar QED and $\sigma = 1/2$ for spinor QED. The vacuum persistence, twice the sum of residues at simple poles of the vacuum polarization, is given by

$$2\text{Im}\mathcal{L}_{\text{eff}} = (-1)^{2\sigma} \frac{(1+2\sigma)(qE)}{2\pi} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \ln\left(1 + (-1)^{2\sigma} \mathcal{N}_{\mathbf{k}}\right), \quad (2)$$

where the mean number of produced pairs and the inverse temperature from the Unruh effect [6] are

$$\mathcal{N}_{\mathbf{k}} = e^{-\beta(\frac{\mathbf{k}_\perp^2}{2m} + \frac{m}{2})}, \quad \beta = \frac{2\pi}{(qE/m)}. \quad (3)$$

The inversion of spin-statistics has been argued in the vacuum polarization [7, 8] and in the vacuum persistence [6], but its physical origin and meaning has not been understood yet. The Schwinger limit is the critical strength for e^-e^+ pair production, $E_c = m^2/|e| = 1.3 \times 10^{16}$ V/cm.

In the in-out formalism the Schwinger variational principle leads to the effective action [4]

$$e^{iW} = e^{i \int d^D x \sqrt{-g} \mathcal{L}_{\text{eff}}} = \langle 0, \text{out} | 0, \text{in} \rangle. \quad (4)$$

The effective action (4) is equivalent to summing the Feynman diagrams in Figure 1. The pair production necessarily makes the effective action complex since $|0, \text{out}\rangle \neq |0, \text{in}\rangle$. Further, the vacuum persistence and the mean number of produced pairs are related through

$$e^{-2\text{Im}W} = |\langle 0, \text{out} | 0, \text{in} \rangle|^2, \quad 2\text{Im}W = (-1)^{2\sigma} VT \sum_{\mathbf{k}} \ln[1 + (-1)^{2\sigma} \mathcal{N}_{\mathbf{k}}]. \quad (5)$$

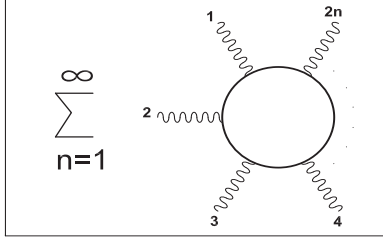


Figure 1: One-loop diagrams: the internal loop denotes a charged particle and the external legs (wave lines) denote the background photons and/or gravitons.

In the above $2\text{Im}W/(VT) = 2\text{Im}\mathcal{L}_{\text{eff}}$ is the decay-rate of the in-vacuum per unit volume and per unit time and for a small pair-production rate, $2\text{Im}\mathcal{L}_{\text{eff}} \simeq \sum_{\mathbf{k}} \mathcal{N}_{\mathbf{k}}$.

Recently Kim, Lee and Yoon have further developed the in-out formalism and introduced the gamma-function regularization (Γ -regularization) [9, 10, 11]. The zero-temperature effective action for bosons and fermions is given by

$$\frac{W}{VT} = \mathcal{L}_{\text{eff}} = (-1)^{2\sigma} \sum_{\mathbf{k}} \ln \alpha_{\mathbf{k}}^*. \quad (6)$$

Here $\alpha_{\mathbf{k}}$ is the Bogoliubov coefficient between the out- and the in-vacua for each quantum number \mathbf{k}

$$\hat{a}_{\text{out},\mathbf{k}} = \alpha_{\mathbf{k}} \hat{a}_{\text{in},\mathbf{k}} + \beta_{\mathbf{k}} \hat{a}_{\text{in},\mathbf{k}}^\dagger, \quad (7)$$

and the coefficients satisfy the relation from the spin-statistics theorem

$$|\alpha_{\mathbf{k}}|^2 + (-1)^{2\sigma} |\beta_{\mathbf{k}}|^2 = 1. \quad (8)$$

The mean number of produced pairs in (5) is given by $\mathcal{N}_{\mathbf{k}} = |\beta_{\mathbf{k}}|^2$. In a constant electric field, the Bogoliubov coefficient may be found from the spin-diagonal component of the Dirac or the Klein-Gordon equation

$$\alpha_{\mathbf{k}} = \frac{\sqrt{2\pi}}{\Gamma(-p)} e^{-i(p\pm 1)\frac{\pi}{2}}, \quad p = -\frac{1}{2} \mp \frac{i}{2\pi} \mathcal{S}_{\mathbf{k}}, \quad (9)$$

where the upper (lower) sign is from the time-dependent (Coulomb) gauge and $\mathcal{S}_{\mathbf{k}} = (m^2 + \mathbf{k}_\perp^2 - 2i\sigma qE)/(2qE)$ is the instanton action [10].

Table 2 summarizes the background fields in which the pair production and/or the effective actions have been known. The in-out formalism has proved a consistent and computationally powerful method for the effective action and/or the pair production for an electromagnetic field in a curved spacetime such as de Sitter (dS) space or anti-de Sitter (AdS) space. Since the Bogoliubov coefficients can be derived from the exact solution of the field equation, it is expected

Table 2: Exact Effective Action and/or Pair Production

Background Fields	EA and PP	Reference
Constant EM-field	EA and PP	Heisenberg-Euler [1] Schwinger [2]
Sauter-type E-field	PP	Nikishov [12]
Sauter-type E-field	EA and PP	Dunne-Hall [13] Kim-Lee-Yoon [9, 10]
E-field in dS and AdS space	PP	Kim-Page [14] Kim-Hwang-Wang [15]
dS space	EA and PP	Kim [16]
EA: effective action	PP: pair production	

that the effective action may be found when the background field and/or the spacetime have certain symmetry, leading to the exact solution. For instance, the Dirac or the Klein-Gordon equation in a constant electric field has the spectrum generating algebra $SU(1,1)$ and dS and AdS spaces have the maximal symmetry of the given dimensions.

3 Vacuum Polarization and Hawking Radiation

The Hawking radiation of bosons and fermions from a charged rotating black hole is given by [17]

$$N_J(\omega) = \frac{1 - |R_J|^2}{e^{\beta(\omega - m\Omega_H - q\Phi_H)} + (-1)^{2\sigma}}, \quad \beta = \frac{1}{k_B T_H}, \quad T_H = \frac{\kappa}{2\pi}. \quad (10)$$

Here R_J is the amplification factor, Ω_H the angular momentum of the hole, Φ_H the electric potential and κ the surface gravity on the event horizon. In the case of the zero amplification factor, the vacuum persistence is

$$2\text{Im}W = -(-1)^{2\sigma} \sum_J \ln(1 - (-1)^{2\sigma} e^{-\beta(\omega - m\Omega_H - q\Phi_H)}). \quad (11)$$

Note the change of sign in contrary to the QED case.

A four-dimensional Schwarzschild black hole with mass M has the inverse temperature $\beta = 8\pi M$. Denoting $J = \{\omega, l, m, p\}$, with the spherical harmonics l, m and the polarization p and the energy ω , the Bogoliubov coefficients for a massless boson field are found [4]

$$\alpha_J = A_J e^{2\pi M\omega} \Gamma(1 + i4M\omega), \quad \beta_J = -A_J e^{-2\pi M\omega} \Gamma(1 + i4M\omega). \quad (12)$$

Now the effective action (6) takes the form

$$W = i(8\pi M) \sum_l (2l+1)(2p+1) \int \frac{d\omega}{2\pi} \ln \Gamma(1 - i4M\omega). \quad (13)$$

Employing the Γ -regularization, we find the effective action per unit horizon area [5]

$$\mathcal{L}_{\text{eff}} = -\frac{1}{16\pi M} \sum_l (2l+1)(2p+1) \int \frac{d\omega}{2\pi} \mathcal{P} \int_0^\infty \frac{ds}{s} e^{-4M\omega s} \left[\frac{\cos(s/2)}{\sin(s/2)} - \frac{2}{s} \right]. \quad (14)$$

It is remarkable that the effective action (14) and the vacuum persistence (11) have the form (1) and (2) of spinor QED in a constant electric field.

The vacuum persistence quantifies the decay rate of the vacuum due to the Schwinger mechanism or the Hawking radiation. Further, it is known that the trace anomalies explain the vacuum persistence, that is, the Schwinger mechanism and the Hawking radiation. In fact, the vacuum persistence for bosons per unit horizon area [5]

$$2\text{Im}\mathcal{L}_{\text{eff}} = \sum_l (2l+1)(2p+1) \frac{\pi}{12} \frac{1}{\beta^2}, \quad (15)$$

is equal to the total flux from the gravitational anomalies [18].

4 Conclusion

We have presented the one-loop effective action for QED in a constant electric field and the Hawking radiation of a Schwarzschild black hole in the in-out formalism. It consists of the vacuum polarization and the vacuum persistence responsible for pair production. The prominent feature of the nonperturbative effective action for a Schwarzschild black hole is that it shares many features in common with spinor QED effective action in a constant electric field.

There remain a few questions to be further pursued: firstly, to find the local effective action outside the horizon, secondly, to investigate the amplification (grey body) factor, and thirdly, to find the effective action at two-loop and higher loops. Still another interesting question is the Schwinger effect in a Reissner-N rstrom black hole. Finally, the origin of spin-statistics inversion of QED differently from gravity challenges a further study [5, 6, 7, 8].

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